## THE PROBLEM OF CONTROLLABLE

## THERMOPROTECTIVE COATINGS

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The concept of a controllable thermoprotective coating as a structural element under dynamic equilibrium is analyzed and discussed. General conditions (applicable to any mechanism of heat and mass transfer) of stabilization are established. Examples are considered involving certain mechanisms of heat and mass transfer. Test results are also shown.

The deisgn of structures for long operation under exposure to high-temperature, high-velocity, dusty, and chemically aggressive gases is made difficult on the one hand by the inadequate stability of engineering materials and, especially, of those used for protective coatings [1] and on the other hand the high stability of precipitates for the removal of which no effective method has been developed yet [2]. The crux of the matter is that stability of a material is ensured only by a strong interlinkage between its particles and that, consequently, their breakdown is irreversible, while the breakdown of a precipitate is a reversible process and its layers are maintained mainly by replenishment from the ambient atmosphere.

The gist of the proposed composite solution to the problem is that physical processes which lead to the formation of deposits, and which are different from the main operating process, be guidedly used for producing controllable stabilized thermoprotective coatings under dynamic equilibrium [3]. The thickness of such coatings  $\delta$  is maintained within a given range of values by varying the direction (the sign) of the resultant mass current j, which is coupled through negative feedback to the deviation of the coating thickness from its design value  $\delta_d$  in accordance with the following control relations:

$$j \begin{cases} > 0 \text{ when } \delta < \delta_{\mathbf{d}} \\ < 0 \text{ when } \delta > \delta_{\mathbf{d}} \end{cases},$$
(1)

or

$$j\left(\delta_{\mathbf{d}}\right) = 0,\tag{2}$$

$$\frac{dj}{d\delta} < 0. \tag{3}$$

The resultant mass current j is a function of the parameters of the ambient medium  $z_i$  (i = 1, 2, ..., n) and of the temperature of the outside coating surface  $T_c$ , which in turn depends on the conditions of heat transfer and on the coating thickness  $\delta$ . Disregarding the heat capacity of the coating and the thermal flux along its surface, we obtain the following system of equations of simultaneous heat and mass transfer:

$$\frac{-d\delta}{dt} = \frac{j}{\rho} , \qquad (4)$$

$$j = j (T_{\mathbf{c}}, z_i), \tag{5}$$

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$$q(T_{\mathbf{c}}, z_i, j) = \frac{\lambda}{\delta} (T_{\mathbf{c}} - T_{\mathbf{b}}).$$
(6)

The thickness  $\delta$ , which may be stipulated in terms of design values, and the possible means of maintaining it are determined by the form of the solution to system (4)-(6). On this basis, there are two possibilities: when

$$\frac{dj}{d\delta} \neq 0,$$
(7)

and the special case when

$$\frac{dj}{d\delta} \equiv 0. \tag{8}$$

In the general case (7), system (4)-(6) is a closed one and has a solution in the form of functions  $\delta = \delta(t, z_i)$ ,  $T_c = T_c(\delta, z_i)$ ,  $j = j(\delta, z_i)$ . If  $j(\delta, z_i)$  is continuous in  $\delta$  and has the root  $\delta_e(z_i)$  in this variable, then with the initial condition

$$\delta_{t=0} = \delta_0 \lesssim \delta_e \tag{9}$$

system (4)-(6) has the following solution:

$$\delta = \begin{cases} \delta(t, z_i) \leq \delta_e & \text{at } t < t_e, \\ \delta_e & \text{at } t \geq t_e, \end{cases}$$
(10)

$$j = \begin{cases} j(\delta, z_i) \ge 0 & \text{at } t < t_e, \\ 0 & \text{at } t \ge t_e, \end{cases}$$
(11)

$$T = T \quad (s = s) \tag{12}$$

$$T_{\mathbf{c}} = T_{\mathbf{c}} \quad (\mathbf{0}, \ \mathbf{z}_i). \tag{12}$$

It follows from system (10)-(12) that only  $\delta_e$  can be the design value  $\delta_d$ . The control of a coating in this case consists in selecting the values of parameters  $z_i$  of the main process which will ensure the desirable thickness  $\delta_e$  (operational regulation) and also in affecting  $\delta_e$  by a proper variation of parameters  $z_i$  (combined operational-forced regulation).

In the special case (8), system (4)-(6) is not a closed one and conditions (1)-(3) – if realizable – are satisfied at any desired value of  $\delta_d$ . It is sufficient here that for a certain  $z_k$  the following be valid:

$$j(z_k) \begin{cases} > 0 & \text{when } z_k \subset \Phi, \\ < 0 & \text{when } z_k \subset \Psi. \end{cases}$$
(13)

Then the given value of  $\delta_d$  is forcibly maintained by acting on j through variation of the parameter  $z_k$ , which is coupled to the deviation of  $\delta$  from  $\delta_d$  through negative feedback.

Without detracting from the generality of the results, we will limit this analysis to the case of convection with a constant coefficient of heat transfer in the gas  $\alpha$ . The thermal flux is then

$$q = \alpha \left( T_{\mathbf{s}} - T_{\mathbf{c}} \right), \tag{14}$$

which, after insertion into (6), reduces system (4)-(6) to the following dimensionless form:

$$\frac{d\operatorname{Bi}}{d\tau} = I,\tag{15}$$

$$I = I(\Theta, z_i), \tag{16}$$

$$\Theta = (1 + Bi)^{-1}.$$
 (17)

Relations (7)-(13) cover the dimensionless expressions for the basic quantities: the coating thickness (Bi =  $\delta \alpha / \lambda$ ), the temperature difference [ $\Theta = (T_s - T_c) / (T_s - T_b)$ ], the mass current (I =  $j/j_{char}$ ), and time ( $\tau = t\alpha j_{char} / \rho \lambda$ ).

We will now consider a few examples pertaining to certain specific mechanisms of mass transfer.

Example 1. When liquid aerosol precipitates and solidifies on a barrier which in this case is a blade of a cooled gas turbine, then the local mass current due to inertia of particles [4] and thermophoresis [5] can be described by the equation:



Fig. 1. Dimensionless dynamic characteristics of a controllable thermoprotective coating with a melted surface at  $\Theta_0 = 1$ : a)  $\Theta(b, \tau)$ ; b) Bi(b,  $\tau$ ); 1) with b = 0; 2) 1; 3) 100; 4) limiting value at  $\Theta = \Theta_{m}$ .

Fig. 2. Heat transfer and equilibrium thickness profile of a controllable coating around the perimeter of a cooled gas-turbine blade: 1) diagram of the heat-transfer coefficient  $\alpha$ ; 2) outer boundary of controllable coating; 3) contact boundary between coating and blade; 4) the same for an erosion-regulated coating on the front edge surface; 5) x-coordinate on the profile perimeter.

$$j = B(z_i, x) + A(z_i) (T_s - T_c).$$
(18)

Letting

 $j_{\rm char} = A(z_i) (T_{\rm s} - T_{\rm h}),$ 

we have

$$I = b(z_i, x) + \Theta, \tag{18}$$

and inserting this into system (15)-(17) together with the initial condition

$$\Theta_{\tau=0} = \Theta_0 > \Theta_m > 0 \tag{19}$$

yields the following parametric solution to the system:

$$\tau + \tau_{0} = f(b, \Theta) = \begin{cases} (b \Theta)^{-1} \left[ 1 - b^{-1} \Theta \ln (\Theta^{-1} b + 1) \right] & \text{for } b \neq 0, \\ 0, 5 \Theta^{-2} & \text{for } b = 0, \end{cases}$$

$$\tau_{0} = f(b, \Theta_{0}), \\ \text{Bi} = \Theta^{-1} (1 - \Theta), \\ I = b + \Theta, \end{cases}$$
(20)

shown graphically in Fig. 1.

According to the graphs and also according to the original equation (18'), this mechanism of mass transfer with I > 0 makes the coating  $\delta$  increase and the temperature difference  $\Theta$  decrease continuously. As a result, however, the temperature difference reaches a value  $\Theta_m$  at some time  $\tau = \tau_m$  which corresponds to  $Bi = Bi_m$  and  $I = I_m$ . After that, the original mechanism of mass transfer becomes augmented by a drain of the liquid film. The mass current across every blade section normal to the flow of the liquid film *l* is then described by the function in [6] (at a constant viscosity of the melt  $\eta_L$ ):

$$\dot{I}_{\rm L} = \int_{t_{\rm L}} \left[ B + A \left( T_{\rm s} - T_{\rm c} \right) \right] dl - \frac{\rho_{\rm L}}{\eta_{\rm L}} \left[ \frac{1}{2} \frac{f}{\delta_{\rm L}} + \frac{1}{3} \left( \mathscr{P} - \frac{dP_{\rm G}}{dl} \right) \right] \delta_{\rm L}^3.$$
(21)

It is evident in (21) that the negative term increases fast (proportionally to the second or the third power) as the thickness of the liquid film increases, while the positive term remains almost constant and even decreases slightly when B = 0, because the temperature  $T_c$  rises. Therefore, for every section normal to the flow of molten film the function  $j_L$  has a root  $\delta^e_L$ . It is also quite evident that  $dj_L/d\delta_L < 0$  for every  $\delta_L$  and thus also for  $\delta^e_L$ . Disregarding, to the first approximation, the thermal resistance of the molten film, we obtain the local parameter values for a thermoprotective coating under equilibrium conditions:

$$\operatorname{Bi}_{m} = \Theta_{m}^{-1} (1 - \Theta_{m}), \tag{22}$$

$$\delta_{\rm m} = {\rm Bi}_{\rm m} \, \frac{\lambda}{\alpha} \, . \tag{221}$$

The profile (22') is realized when a blade is made according to a diagram with  $\alpha$  already calculated [7], while the outside coating surface duplicates the theoretical contour (Fig. 2). Such an equilibrium distribution was obtained by the authors for a cooled conical diffuser exposed to streams of benzene combustion products with contaminating dust ( $T_s = 1790^{\circ}$ K,  $T_m = 1713^{\circ}$ K,  $\Theta_m = 0.05$ ,  $M_{out} \simeq 1$ ).

Example 2. The precipitation mechanism is diffusive between vapor and the flowing liquid mass, based on the analogy between heat and mass transfer and the Arrhenius law [9]:

$$I = \exp\left(-\varkappa \operatorname{Bi}_{\operatorname{sat}}^{-1}\right) - \exp\left(-\varkappa \operatorname{Bi}^{-1}\right),$$

$$i_{\text{char}} = \frac{\beta}{RT_G} P_B \exp((-\varkappa)) = \frac{\beta}{RT_G} P_{\text{sat}}(T_s),$$

$$\varkappa = E (RT_S)^{-1},$$
(23)

which together with system (15)-(17) and the initial condition

$$\operatorname{Bi}_{\tau=0} = \operatorname{Bi}_{0}$$

yields the parametric solution

$$\tau = \int_{\mathrm{Bi}_{\bullet}}^{\mathrm{Bi}} \left[ \exp\left(-\varkappa \operatorname{Bi}_{\operatorname{sat}}^{-1}\right) - \exp\left(-\varkappa \xi^{-1}\right) \right]^{-1} d\xi,$$
  
$$\Theta = (1 + \operatorname{Bi})^{-1}.$$
(24)

As in the previous example, Bi and  $\Theta$  reach their equilibrium values  $Bi_{dew}$  and  $\Theta_{dew}$  at time  $\tau_{dew}$  but, unlike there,  $I(Bi_{dew}) = 0$  with the stability of a coating ensured within the effectiveness of the given deposition mechanism. The preceding discussion pertaining to the molten film thickness  $\delta_m$  applies also to the equilibrium film thickness  $\delta_{dew}$ .

Example 3. Solid aerosol precipitates and the excess layer is eroded by the inertia mechanism according to the following relation for the mass current [10, 11]:

$$j = M(z_i, x) (1 - \omega) - N(z_i, x) \omega.$$
(25)

The subsequently acting mechanism applies to the case (8) with the concentration of abrasive particles  $\omega$  as the regulating parameter  $z_k$  (the abrasiveness of particles increases with the size of particles, other conditions remaining unchanged). Sets  $\Phi$  and  $\Psi$  are defined by the inequalities:

$$z_h > \frac{M}{M+N} , \qquad (26)$$

for  $\Phi$  and

$$z_h < \frac{M}{M+N}$$

for ¥

By such a mechanism the protective film forms only on the front edge surface [4], while the remaining surface must acquire protection by another mechanism as, for example, one of the earlier mentioned ones. In such a combination the coating on the front edge does not depend on  $\alpha$ : it has no minimum thickness determined by the maximum  $\alpha$  but, instead, can have any desired thickness (Fig. 2).

The list of examples can be expanded and, obviously, will cover a wider range as research on this subject continues. Thus, there appear many possibilities for producing controllable thermoprotective coatings. The general principle of their application is that the control process must be organically tied in with the main operating process of the protected object.

## NOTATION

δ	is the thickness of coating;
ρ	is the density of coating;
λ	is the thermal conductivity of coating;
T <sub>S</sub>	is the stagnation temperature of gas stream;
T	is the surface temperature of coating;
Th	is the surface temperature of base material under coating;
q	is the thermal flux density;
α	is the heat-transfer coefficient;
J. Jchar	are the resultant mass current and its characteristic value:
A	is the proportionality factor between temperature difference and mass current
	due to thermophoresis;
В	is the mass current due to inertial collisions of particles against barrier:
P <sub>B</sub> , E, R	are the base pressure, activation energy, and gas constant of the sublimate.
2	in the Arrhenius equation;
$P_{sat}$	is the partial pressure of saturated vapor at a given temperature;
$\beta, T_{G}$	are the mass-transfer coefficient and characteristic temperature at which it
-	is determined;
ω	is the fraction of abrasive particles in the solid phase of the operating body;
M	is the proportionality factor between fraction of nonabrasive particles in solid
	phase of working body and mass flow to surface;
N	is the proportionality factor between fraction of abrasive particles and the mass
	eroded from the surface;
t	is the time;
z	is the i-th parameter of the two-phase stream;
Φ, Ψ	are the sets of parameter $z_k$ values;
x	is the coordinate in the mainstream direction;
l	is the coordinate in the direction of film flow;
ξ	is the integration variable;
η	is the dynamic viscosity;
f, $\mathcal{P}_{dP_{G}}/dl$	is the frictional stress, mass force, and pressure gradient along operating
	body;
$I = j/j_{char}$	is the dimensionless mass current;
$\Theta = (T_s - T_c) / (T_s - T_b)$	is the dimensionless temperature difference;
$Bi = \delta \alpha / \lambda$	is the dimensionless coating thickness, Biot number;
$\tau = t\alpha j_{char} / \rho \lambda$	is the dimensionless time;
Mout	is the Mach number at exit from the test nozzle.
Subscripts	
e denotes the c	conditions of dynamic equilibrium between coating and operating body (base ma-
terial);	-

	terial);
m	denotes the conditions when the outside coating surface is at the melting temperature;
sat	denotes the saturated vapor;
dew	denotes the conditions when the outside coating surface is at the dew-point temperature of
	the vaporous precipitating admixture;
$\mathbf{L}$	denotes the liquid film of molten coating;
char	denotes the characteristic value.

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